Navigation, Maths & Astronomy: the Pagan Knowledge

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It may sound strange that the adventurous sailors like Columbus and Vasco da Gama were no great navigators? The European method of navigation by 'dead reckoning' necessarily relied upon maps and charts, so they did not know how to navigate on un-charted seas.

Behind this enigma lies the ignorance of maths of the Europeans, and the story of navigation, calendars and clocks.

The methods of timekeeping in Europe, whether through mechanical clocks or the calendar, remained remarkably inaccurate until the 16th century CE, when this became a major embarrassment to both church and state. Unlike India which was far better off, the early 16th century Europe was very poor. The most prosperous regions were only Spain and Portugal, just emerging from Arab rule. Trade with the rich states of India and China represented a golden opportunity. Motivated by abject poverty and the hope of future riches, European sailors were ready to run huge risks: approximately a third of them used to die on each successful voyage to India. Ships sank frequently, and a sunken ship meant also loss of valuable cargo. Ultimately, successful trade needs secure trade routes and secure travel from Europe to India or China and back needed, at the least, knowledge of navigation. Navigation was the strategic and economic key to the initial prosperity of Europe through trade and subsequent colonisation.

Many of us would be surprised to know that Columbus and Vasco da Gama were hardly great navigators, though they certainly were great adventurers. Neither knew the celestial navigation techniques known to their Indian, Arab, and Chinese contemporaries. They had heard of celestial navigation, used by Arab navigators, but did not quite understand it.

Columbus' first recorded attempt at using a quadrant to establish his latitude was on 2 November when he was off the northern shore of Cuba. This sadly erroneous sighting put him on the latitude of Cape Cod. Even so, Columbus failed to recognize this gross error and instead concluded that he was on the mainland of Cathay. This illustrates Columbus's serious incompetence in celestial navigation. Columbus tried the quadrant again on 20 November and came up with the same deplorable result of 42 degrees north latitude, but this time he realized that something was wrong and blamed it on the quadrant which he said was broken and needed repair. How can a quadrant be broken when it has only one moving part and that part is a string with a weight on the end?

Similarly, Vasco da Gama used the services of an Indian pilot, Kanha, to 'discover' the sea route to India. To determine the latitude at sea, the pilot used an instrument, called *kamal* or *rapalagai.* In its simplest form, the instrument consists of a small wooden board and a string graduated with knots. The local latitude is almost the same as the altitude of the pole star, or its angular elevation above the horizon. To determine the altitude of the pole star, the wooden board is held in front of the eye, at an appropriate distance, so that it blocks the portion between the horizon and the pole star, and the distance from the eye is measured. Holding the string between the teeth, and counting the number of knots, one measures the distance. In the Arabic-Malayalam language, the pole star is hence called *kau,* which also means 'teeth'. Vasco da Gama, not understanding the principle of the instrument, thought the pilot was telling the distance with his teeth! He further recorded that he carried back a couple of copies of the instrument to get it graduated in inches! (The instrument involves a harmonic scale, whereas inches refer to a linear scale, so that graduating it in inches is intrinsically impossible.)

Though the Europeans did not know celestial navigation, their own technique of navigation by 'dead reckoning', using maps and charts, was very unreliable. Though a great deal of effort initially went into procuring and making accurate maps, it was eventually understood that, despite accurate maps, the European technique of navigation itself was inaccurate since it required measurement of the speed of the ship. A process called heaving the log measured the ship’s speed: throwing overboard a log tied to a rope, and measuring out the amount of rope taken up in a given period of time. A sailing manual describes how inaccurate this process was, even in 1864:

“if the gale has not been the same during the whole hour, or time between heaving the log, or if there has been more sail set or handed, there must be an allowance made for it, according to the discretion of the officer. Sometimes, when the ship is before the wind and a great sea is setting after her, it will bring home the log; in such cases it is customary to allow one mile in ten, and less in proportion if the sea be not so great; a proper allowance ought also to be made if there be a head sea. In heaving the log, great care should be taken to veer out the line as fast as the log takes it; for if the log be left to turn the reel itself, it will come home, and give an erroneous distance.”

European ignorance of navigation was widely recognised as a major problem, because the immense economic and strategic importance of navigation for Europe was transparent to all. One sunken ship meant not only a fortune gone, but also more men gone than in a typical war of those times. Consequently, governments in Europe not only officially admitted the European ignorance of navigation, from the 16th to the 18th century they did everything possible to find a better technique of navigation. Pedro Nunes, a professor of mathematics at Lisbon and Coimbra, was appointed royal cosmographer in 1529. Philip II of Spain offered a huge prize in 1567, for a reliable technique of navigation. Many European governments over the next two centuries continued this process of offering huge prizes for navigation.

By the mid-16th century, the Europeans had learnt the basic technique of determining latitude by pole-star altitude, and had devised instruments like the cross staff for this purpose, though these simple instruments lacked the sophisticated interpolation techniques of the Indo-Arabic instrument - techniques which came into general use in Europe only after Vernier in the 17th century CE (after whom they are named).

Using the pole star for navigation had two drawbacks. For travelling from Europe to India, it is necessary to cross the equator. As one moves towards the equator in the northern hemisphere, the pole star ceases to be visible above the horizon; there is no similar star in the southern hemisphere. Moreover, the pole star is not at all visible in the daytime.

For navigation during the day, the Indo-Arabic technique of navigation involved measuring solar altitude at noon. Solar altitude, like the altitude of the pole star, can be measured by any device used to measure angles, such as a cross-staff or a quadrant, or anyone of the great variety of instruments that were devised for this purpose. But there was another problem because latitude cannot be calculated so easily from solar altitude. Unlike the pole star, the sun does not stay approximately fixed, but, as all of us know, the sun moves substantially to the north in summer (in the Northern hemisphere), and to the south in winter.

To calculate the latitude from the solar altitude, it was necessary to know the solar declination or its north-south deviation, at the time of measurement. The solar declination varies from day to day. The declination is zero on the days of the equinoxes, and is a maximum on the days of the solstices. Knowing the maximum displacement, hence the average displacement per day, we can calculate the solar declination on any given day, if we know the number of days that have elapsed since the vernal equinox. For example, if we know that the altitude of the sun at noon is 90 degrees, and we know that today is 22 June summer solstice, then we know that our latitude is the same as that of the Tropic of Cancer. If, however, today is 2 July, then we are far off from the Tropic of Cancer. The dates 22 June and 2 July are not meaningful in themselves, unless one has an accurate calendar, which correctly identifies the vernal equinox. So, to calculate latitude accurately from the measured solar altitude at noon it was necessary to have an accurate calendar.

The calendar used in Europe at that time was the Julian calendar, set up by Julius Caesar. Because the Romans found arithmetical calculations difficult, for simplicity in calculation, the Roman calendar had adopted the figure of 365+1/4 days for the length of the year, a figure which was wrong in the second decimal place, leading to an error of one day in a century. The resulting error had piled up over the centuries, so that in the 16th century the Roman calendar was inaccurate by 10 days. This introduced too large an inaccuracy in deducing latitude from measurement of solar altitude at noon. By way of contrast, the text of Bhaskara **I,** written a thousand years earlier, and widely used in India, speaks of corrections due to the change in solar declination from morning till evening! This latter change being about 1/8 of a degree, the error due to the inaccurate calendar amounted to some 3 degrees of the arc! (One must add also the error due to measurement and the error due to inaccurate sine values). For a sailor this was easily the difference between life and death.

Since the inaccurate Roman calendar put European sailors in the 16th century to such an enormous disadvantage, and since navigation was economically so important to Europe, reform of the calendar became imperative. But correcting the calendar involved another problem. The equinoxes represent the zero point of solar declination; so correcting the calendar for navigation meant correcting the date of the equinoxes. But this meant also revising the date of Easter. This was a problem that involved the church: a powerful entity in 16th century Europe, in the heyday of the inquisition. Recall that the date of Easter was the key point on the agenda of the Nicene council, so the date of Easter practically defined the Nicene Creed. Articulating a difference from the Nicene Creed meant being branded a heretic - a dangerous proposition, even for a Newton in Protestant England, a hundred and fifty years later. So strong were the religious feelings in the matter, that the obvious corrections to the defective calendar were not accepted in England until 1752. Discontent with the Roman calendar had been earlier voiced in Europe for several centuries, but had been ignored until the 16th century, when an accurate calendar became a matter of the greatest practical importance to the state. Even after the Roman Catholic Church had publicly accepted the need for a calendar reform, the actual process of reforming the calendar and revising the date of Easter took some 50 years. The calendar reform focused on the date of the equinox, and did not address the obvious absurdity of retaining a calendar with months, unrelated to the natural cycle of the moon, and varying in length from 28 to 31 days. Thus, in the sixteenth century, fixing the date of Easter had again become the major scientific, technological and religious problem of Europe!

The Jesuit Christoph Clavius who eventually headed the calendar reform committee had studied at Coimbra under Pedro Nunes, the most famous European navigational theorist of the time. Clavius reformed the curriculum of Jesuit priests at Collegio Romano, to introduce (practical) mathematics into it, as noted earlier, and he wrote a text on practical mathematics. From among the first batch of Jesuits, so trained in mathematics and navigation, the most capable, like Matteo Ricci, were sent to collect information about timekeeping from India, to help in Clavius' reform of the Gregorian calendar.

The insularity of the church now assumed a new form. Though it privately sought 'pagan' learning, it continued publicly to deny that there was any learning among the 'pagans'. It needed, therefore, to hide its dependence on pagan learning for so central a religious festival as Easter. Thus, though Matteo Ricci visited Cochin, a centre of Indian *jyotisa* (timekeeping through astronomy and mathematics), in 1581, and himself wrote that he was trying to learn about the methods of reckoning time from 'an intelligent Brahman or an honest Moor', the *Encyclopaedia Britannica CD97* still records that 'Matteo Ricci was sent to Cochin for reasons of health'!

Indeed, Western historians, especially from the 18th to the 20th century, have spent much effort to show the irrelevance of 'pagan' learning. The claim is that the present stock of knowledge is entirely free of any corrupting 'pagan' influence. The classical trajectory of knowledge development, still widely prevalent today is:

Greece →Renaissance →Modern Science

According to this trajectory, no theologically incorrect part of the world has played any mentionable role in the development of knowledge. It is now beginning to be recognised that, for example, this trajectory needed to fabricate ancient Greece, through appropriation of African learning. It bypassed Indian and Arabic learning. Copernicus' heliocentric model, for instance, was but a bad Latin translation of a Greek translation of an Arabic work on astronomy. This very strange current-day belief that only Christians, or their theologically correct predecessors in Greece have developed almost all-serious knowledge in the world, demonstrates the strength of the continuing cultural feeling against 'pagan' learning. There is nothing 'natural' or universal in hiding what one has learnt from others: the Arabs, for instance, did not mind learning from others, and they openly acknowledged it. This is another feature unique to the church: the idea that learning from others is something so shameful that, if it had to be done, the fact ought to be hidden. Therefore, though the church sought knowledge about the calendar, specifically from India, and profusely imported astronomical texts (the Jesuits, of course, knew the languages of these texts, and had even started printing presses in some of these languages by then), this import of knowledge remained hidden.

Longitude

After Pope Gregory's Bull of 1582, which reformed the Roman calendar by adding ten days to the calendar, on October 5, and introduced the system of bypassing leap years every century, the problem of determining the latitude at sea was solved. But the navigational problem persisted, because longitude could not be accurately determined!

The navigational knowledge of determining local latitude and longitude, that the Europeans sought, existed, for example, in widely distributed Indian calendrical manuals from the 7th century, such as the texts of Bhaskara. This knowledge had been revised and updated over the centuries, by various people including Al-Biruni in his famous treatise on mathematical geography, and a prominent school of mathematics in Kerala. This revised and updated knowledge was recorded in calendrical and astronomical manuals widely distributed in the vicinity of Cochin, where Matteo Ricci and other Jesuits searched for them. Language was not a barrier, and after Clavius, knowledge of mathematics was also not a barrier. Ironically, however, this navigational knowledge in Indian and Arabic texts could not be used directly by the European navigators because of some other difficulties.

The first difficulty was still the same old inability to calculate. Though the experts in Europe were beginning to learn about the decimal representation, and know, by then how to use algorithms to add, subtract, multiply, and divide, they did not thoroughly understand the calculus and trigonometry. Trigonometry came to Europe, after Regiomontanus, at least a thousand years after it had developed in India. European errors in understanding trigonometry are embedded in the very names of the trigonometric functions! Thus, the Indian term for the sine was *jya* or *jiva.* This was taken into Arabic as *jiba.* However, Arabic writing often omits vowels, so the term *jiba,* written simply as *jb,* was misunderstood as *jmb* or fold, and translated into the Latin *sinus* ! Calculus was needed to derive precise values of the sine function, which were available in contemporary 16th century Indian texts like the *Tantrasangraha* and *Yuktibhasa.* Key figures of the time in Europe, such as Pedro Nunes, Christoph Clavius, and Simon Stevin, all published texts containing tables of the sine function and other trigonometric functions useful in navigation, and tried to make their tables as accurate as the contemporary Indian tables. The sine function was involved in determining latitude. It was also involved in Bhaskara's method or Al Biruni's method of determining longitude from knowledge of the latitude difference together with some other information.

The calculation techniques in India had advanced substantially beyond the algorithms for multiplication and division, and decimal fractions that Europe was just beginning to get used to in the late 16th century CE. Though right from the time of Christoph Clavius, and the calendar reform of 1582, active efforts were being made to procure calendrical and mathematical knowledge from Indians, Arabs, and Chinese, Europeans had difficulty in understanding these texts. The results of this import of mathematical and astronomical knowledge is reflected in the work of the 17th century European mathematicians like Cavalieri, Fermat, Pascal, and Gregory, directly, and

Leibniz, Wallis, and Newton, indirectly, though they did not mention their sources, and, often did not reveal their methods. Fermat's famous challenge problem to European mathematicians, for instance, is found as a solved problem in several popular Indian astronomical and mathematical works, including those of Brahmagupta and Bhaskara II. Nevertheless, leading European mathematicians had fundamental difficulties in understanding these imported techniques of calculation, involving infinite series, which Descartes declared to be beyond the capacity of the human mind. These difficulties were natural, for the traditional Indian understanding of mathematics as practical, computational, and empirical, contrasted sharply with the European understanding of mathematics as spiritual, proof-oriented, and formal. In the *Yuktibhasa* derivation of the infinite series, in accordance with the *Nyaya-Vaisesika* philosophy of atomism, the process of subdividing a circle was presumed to stop when the subdivisions reached atomic proportions. But when the Jesuit Cavalieri used the term 'indivisible', while similarly deriving the same infinite series, this led to a storm of protest. These difficulties with the infinitesimal calculus persisted in Europe until the late 19th century CE.

The size of the globe was another important piece of information that went into the Indo-Arabic methods of determining longitude. Lacking an accurate knowledge of the size of the globe, Europeans could not use these methods in the 16th century and for much of the 17th century. Indians and Arabs had determined the size of the globe very accurately. The methods ranged from the inexpensive techniques documented by Al-Biruni, to that of Caliph al Mamun, who sent an expedition in the desert to physically measure out the distance of one degree of the arc. Though Europeans were presumably aware of the earlier Indo-Arabic estimates, the irony was that Columbus, perhaps to get finance for his voyage, had understated the size of the globe by 40 per cent. Columbus' 'success' seemed to confirm the estimate, so that few people cared to revise it! Instead, Portugal banned the use of the globe for navigation, despite Nunes' valiant attempts to defend it. Ultimately, when Newton did suggest a revision of the size of the earth, he was still 25 per cent below the mark.

By this time (mid-16th century CE), the navigational problem had assumed such acute proportions that the state started intervening more and more actively to encourage the development of a solution. The reward offered by Philip had been increased in 1598. The reward was now so large that the most prominent scientists of the time competed for it. Galileo, for example, tried to get the reward for nearly 16 years, starting in 1616. After that he shifted his attention to the prize offered by the Dutch government in 1636. In France, Colbert, following his predecessors Mazarin and Richelieu, offered vast sums of money for a solution to the navigational problem, and sent personal invitations to Huygens, Leibniz, Roemer, Newton, Picard etc to tackle it. From the replies he received, he selected 15 people to form the French Royal Academy.

The British Royal Society was started similarly, around groups which met to discuss the 'longitude problem'. A 1661 poem describing the work going on at one of these groups at Gresham College went as follows:

The College will the whole world measure,
Which most impossible conclude,
And Navigators make a pleasure
By finding out the longitude.
Every Tarpalling shall then with ease
Sayle any ships to th'Antipodes.

(Tarpalling here means a tar or a sailor.) The group from Gresham College included John Wallis and Robert Hooke; it later merged with other groups to form the Royal Society of London. Christopher Wren, also a member of the Gresham College group, wrote the preamble to the Royal Society's charter. One of the stated aims of the newly founded Royal Society was: 'Finding the Longitude.'

As the first project of the French Royal Academy, Picard re-determined the size of the earth in 1671, using Caliph al Mamun's technique of physically measuring one degree of the arc. For longitude, Picard's method used the same principle of timing eclipses that was used earlier by Bhaskara and Al-Biruni. This principle provided an operational definition of simultaneity between physically separate locations, enabling one to measure the difference of local time between these locations. Picard's method, however, was adapted to the improved technology of the telescope, following a suggestion by Galileo, to use the eclipses of the moons of Jupiter. This enabled the first European determination of longitude on land.

The Chronometer and Navigation

The Europeans, however, continued to have difficulties with determining longitude at sea - while at sea it was then (before the radio) not possible to compare notes with a distant observer. It was for this navigational problem that the mechanical clock was first put to practical use, instead of ritual use, so that its accuracy became significant from a practical point of view. The development of the mechanical clock not only provided a powerful metaphor for the development of a mechanical society, the mechanical clock is a serious contender with the steam engine as a symbol of the industrial revolution. Navigation using the mechanical clock revolutionised shipping even before railways could revolutionise overland transport.

Strictly speaking, a mechanical clock was not an essential prerequisite to the industrial society. After Picard's measurement of the size of the earth, and following the import of the calculus and precise sine values into Europe, it was possible for Europeans to have shifted to the Indo-Arabic techniques of celestial navigation. However, this would have required sailors to do advanced mathematical calculations in their head, and so would have required a transformation of the educational system which remained the preserve of priests and the aristocracy. Considering that Britain had, by then, not yet accepted the reformed calendar, it was easier to develop the mechanical clock than to transform the society, by changing the educational system.

What has a clock got to do with longitude? Imagine that you are stranded in the Sahara desert. Let us say that, inspired by an amateur geological theory, you charter a flight to make an aerial survey of the seif dunes. The plane develops a fuel leak, and you are forced to land in the midst of a sand sea. You have just enough time to scramble out before the plane catches fire and explodes, killing the pilot. What should you do? The best thing is to sit near the debris of the plane and wait for a rescue party. An hour passes. The sun is very hot; you are thirsty. Another hour passes. You are weak with thirst. The rescue party had better come soon.

Suddenly you see a slight movement on the horizon. Is that a mirage? No. It is an approaching sandstorm. The air is clear; there is no dust; yet a vast quantity of sand is moving. You hide behind a rock, and wait for the sandstorm to pass. You survive, but the debris of the plane is completely buried under the sand. Nothing of the plane is now going to be visible from the air. No rescue party for you.

But you don't give up. You start thinking. You have thoroughly studied the area you proposed to survey. You have a map of it in your head. There are two oases nearby. But both are isolated. You must move in practically the exact direction towards an oasis. If you make a mistake, you will probably die of thirst before you find the oasis. Desperation sharpens your mental faculties. You can see very clearly the exact details of the map in your head. The best thing would be to travel during the night. (You are also an amateur astronomer, and have studied all about the ancient technique of navigating by the stars). To make things a little easier for you, we will suppose that both oases lie exactly along an easy-to-identify stellar rhumb line.

But a new difficulty now arises. The two oases are far apart. If you can reach one, you can't reach the other. In which direction should you move? You must decide quickly; time is passing, and each passing moment makes you thirstier. Involuntarily you glance at your wristwatch. And you discover the mistake that saved your life. When you landed at the airport on the regular flight from Delhi, you forgot to correct your watch. It still shows Delhi time. You stick a pen vertically into the sand, and start marking the time against the tip of its shadow. When the shadow is shortest, the sun is as vertically overhead as it can get: so that locally it is noon. Comparing this with your watch tells you the time difference, hence the longitude relative to Delhi. (Each 4 minutes gain equals a degree of longitude, since 24 hours' equals 3600 of longitude.) Having made your calculation you settle down to wait for the evening. A quick glance at the setting sun, a few finger measurements with the rising stars, a short mental calculation, and you are confidently on your way.

Though the method of determining longitude from time difference was well known to Bhaskara I, your technique of navigating by the *mechanical* clock would have been unavailable to a 17th century traveller lost in the desert. Though the mechanical clock existed, it was neither portable nor accurate enough for this purpose. In fact, in the 17th century, Europe had still not learnt any reliable technique of navigation. Europeans still knew of no reliable way of determining longitude at sea, though ships used to travel great distances. Following some spectacular maritime disasters in 1707, Isaac Newton deposed before a Parliamentary committee formed to look into the matter:

That for determining the Longitude at Sea, there have been several Projects, true in theory, but difficult to execute. One is a Watch to keep Time exactly, but.. .such a Watch has not yet been made.

There were several difficulties in making such a watch. For example, it had to be miniaturised, so that it could be easily carried aboard a ship. It had to be made immune to the constant motion of a ship, and immune even against the rolling of the ship during a storm - it had to be made 'shock proof'. It had to be made immune to variations in temperature, and humidity; 'waterproof' was the least the Watch had to be.

A bill was soon approved to provide a reward of £20,000, and a Board of Longitude was formed. Supported by the Board from 1735 onwards, John Harrison eventually produced the required mechanical watch, which easily passed the stipulated test on a voyage to Jamaica in 1757. By the mid-nineteenth century, the chronometer had become reliable enough to come into widespread use. The West had finally picked up a lead in technology over the East. The watch in this miniaturised and carefully standardised form, used as an instrument for navigation, came to be called the chronometer.

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