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Axiomatism and Computational Positivism

Two Mathematical Cultures in Pursuit of Exact Sciences

It is argued here that the mathematical approach to the exact sciences has historically appeared to contain two largely distinct cultures (which nevertheless overlap to some extent). One of these takes the deduction of 'certain' conclusions from clearly stated axioms or models as the primary objective; the other considers number the primary concept, and emphasises computation and algebra, conforming to unambiguous rules. A philosophy that may be called computational positivism, whose goal is to make computation agree with observation, appears to have been characteristic of Indian (and apparently Babylonian) astronomy. The interactions between these two cultures have played a key role in the history of science, and seem set to continue to do so in the future as well.

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I- Introduction

Positivism was a great philosophical movement that emerged in Europe in the second half of the 19th century. Its roots have been traced to Francis Bacon (1561-1626) and the English empiricist school; it held that facts are the only possible objects of knowledge and science the only valid knowledge. It generally opposed any kind of metaphysics. Many different kinds of positivism developed later, but in the first half of the 20th century the famous Vienna Circle (consisting of scientists, mathematicians and philosophers) gave birth to 'logical' positivism. One of the central tenets of this school was verifiability - a statement, which cannot be verified, was automatically held to be meaningless. There were only two types of meaningful statement: the necessary truths of logic, mathematics and language, and empirical propositions about the rest of the world. In particular Wittgenstein (although not formally a member of the Circle) argued that the propositions of logic and mathematics are tautologies - and certainly not truths of a higher order than those based on experience, as classical Western rationalists had held. Although the Vienna Circle broke up in the 1930s and logical positivism has had its detractors and ceased to be a major - movement, some of its ideas continue to 'influence many scientists even today.

As I have briefly argued elsewhere (Narasimha 2003) one can identify one other kind of positivism, and that may legitimately be called 'computational'. This philosophy may loosely be said to hold that computation and observation, when the two agree with each other, constitute the only form of valid knowledge; models, logic, metaphysics etc. are either secondary or not relevant. Models may *not* be unique (in the sense that different models may yield very similar results in a domain of interest) and logic (as Wittgenstein pointed out) is tautology. This attitude, often implicitly and occasionally explicitly, informed the classical Indian mathematical approach to astronomy; recognition of this fact illuminates the view generally taken of exact science in Indian civilisation (and possibly some other non-western ones as well). The purpose of the present article is not to defend or propagate computational positivism, but rather to identify and describe it and what (it is hoped) will emerge as its chequered history - a history that in the present view is not yet complete.

The subject of mathematics is often seen as the most nearly universal of intellectual disciplines. Nevertheless, all practising scientists know that, even today, the mathematics done in any country, even within the western cultural area, has its own special character (e.g., British, French, Russian etc). This does not appear to be just a matter of style, but rather of philosophy: of the questions asked, and the manner in which they are tackled. Across civilisations the differences can be even stronger, as I hope this paper will show. But it is necessary to point out that this is not to take a relativist view of science; each approach is objective in its own way, in the sense that anybody following the processes of any of the different systems - together with their explicit or implicit assumptions - will come to the same conclusions. All are rational, but some of course may be more effective than others according to some particular criterion, such as, e g, degree of agreement with observation, economy of thought or process, the extent of phenomenological domain covered, or even notions of beauty or symmetry; but the criterion itself may vary from one system to another.

We shall go back in history illustrate this idea by a comparison between classical Hellenistic and Indic astronomy.

II- A cross cultural comparison

Astronomy is an excellent candidate for such comparison because the motion of objects in the heavens, in particular planets, is something that can be observed virtually in any part of the globe; the basic data are thus the same in all civilisations, and not greatly affected by their geographical location. Furthermore there is clearly some regularity or order in the motion of the planets in the heavens, but at the same time to an observer on the earth the motions are not entirely simple; for example, planets can exhibit retrograde motion some times, i.e., seem to backtrack in their orbit. All great civilisations in the world have tried to reduce these motions to some more or less quantitative system, but the approaches they have taken are not the same. A comparative exercise of the present kind may therefore help to reveal the basic predilections of different cultures in organising reality through quantitative observation and predictive mathematics of some kind.

The history of astronomy is ancient, but we will choose for comparison the two great astronomers Ptolemy (2nd century CE, Hellenist) and Aryabhata (5th century CE, Indian, Shukla and

Sarma 1976). Ptolemy's work can be seen as the culmination of the Hellenist enterprise in astronomical prediction, for his work was not improved upon in the West till after Newton some fifteen centuries later. But the special point of interest for us here is that the way that Ptolemy precedes about his task represents a very definite view of how science is to be done. His great treatise has been generally called the *Almagest* - incidentally a name that reveals Arabic connections, as this text, like many others, had to be recovered in medieval Europe from an extant Arabic translation. The text consists of 13 books of which the first sets out in detail a 'model' and the assumptions ('conceptions') that underlie it. These assumptions include statements about how the heavens are spherical and move spherically, and how the earth is also sensibly spherical and is in the centre of the heavens; the earth has no local motion at all. Because the Greeks thought of the circle as a perfect figure, planets tended to move in circles, but retrograde motions of the type mentioned above showed that a single circle would be inadequate; so Ptolemy is compelled to represent such motions by epicycles, i.e., circles whose centres move on other circles. All these assumptions are justified in considerable detail, and the rest of the book is an attempt to deduce planetary motions solely on the basis of the model set out in the first book. There were earlier Greek models, in particular involving what has been called a homospheric system. In these models the universe was finite; planets moved in spherical shells made of crystal. This view was necessary because the Greeks had philosophical objections to the notion of vacuum (recall that according to Aristotle nature abhors vacuum), but the necessary space-filling material had to be transparent in order for one to be able to see the 'fixed' stars, lying on a (finite) sphere beyond the planets. The remarkable thing of course is that based on these elaborate assumptions, most of which we now know to be erroneous, Ptolemy could make quite an extraordinary number of predictions whose accuracy would not be surpassed in Europe till some time after Newton.

We see here incidentally that a basically false model may be able to give rather good results.

It is interesting to see how Aryabhata proceeds to tackle the same problem. First of all his book the *Aryabhatiya* (composed 499CE) is very short; it has only 121 slokas, which can all be written on three A4 size sheets of paper. The treatment is very terse, and even cryptic, so it can be understood only with instruction from a guru or a commentator. But what is of interest to us here is that the text is completely different in character from that of Ptolemy or the other Greeks. This is particularly revealing because some of the basic ideas, such as the use of epicycles, seem to have been borrowed from the Greeks. However the epicycle is only used as a convenient representation of motion; there is no mention of a physical or kinematic model, therefore no justification, and certainly no notion that the circle is a perfect figure and must therefore form a necessarily correct description for planetary orbits. Indeed the *Aryabhatiya* is best seen as a collection of more than 50 algorithms, which are basically sets of instructions about calculations to be carried out. (The word algorithm is derived from the 9th century Iranian astronomer mathematician Al-Khwarizmi, who was intimately familiar with Indian mathematical ideas and methods of calculation; his book *Al-Jam'a wal-Tafreeq bit Hisab al-Hindi* provided an important source of ideas and thought to Europe in following centuries.) Aryabhata starts with an introduction in which he describes an ingenious system for expressing numbers. This is followed by lists of various numerical parameters required in the astronomical algorithms that fill the rest of his book. He does have physical concepts, for he talks about how eclipses are basically caused by shadows, realises the significance of relative motion, and proposes that the earth spins on its axis (therefore it is not stationary as in Ptolemy's model). However he does not justify the procedures he advocates or explain their importance in any detail, and makes no attempt to formulate a picture of the motions in the heavens, for here the really important thing is an actual method of calculation of planetary parameters. He is looking for short, effective methods of calculation rather than for a basic model from which *everything* can be deduced. But in formulating the algorithms and his methods of calculation he is ingenious; of this several examples can be quoted including the way he expresses the trigonometric functions, the procedures he describes for solving equations etc. We could in fact legitimately say that he is trying to describe algorithmic or (to use a more fashionable current word) computational astronomy.

The philosophy underlying this approach to astronomy is most explicitly described in the later work of what has come to be known as the Kerala School. This was a group of astronomers and mathematicians who, over a period of some three centuries, produced some very innovative and powerful mathematics applied to astronomy. The main goal of Indian astronomical schools has always been the achievement of *drg.ganitaikya,* which literally means the identity of the seen and the computed. The effort therefore was to find algorithms or computational procedures, which made the best predictions as determined by comparison with observation. One of the leaders of this school, Nilakantha (1444- 1545 CE), declared that 'logical reasoning is of little substance, and often indecisive' - words that would seem to go totally contrary to the approach that was used in Hellenist schools, which followed the Euclidean method of going from well stated axioms through a process of purely logical deduction to theorems or conclusions. The Indian schools were driven by the need to develop the 'best' algorithms, for they noted that, over a period of time, discrepancies between computation and observation tended to increase. Explicit statements are therefore made about how the best mathematicians have to sit together and decide how the algorithms have to be modified or revised to bring computation back into agreement with observations.

It is this attitude that I have called computational positivism elsewhere. The philosophy places computation and observation at the forefront. Elaborate physical models and a process of deduction based on axioms are not seen as of great value; indeed the distrust of deductionist logic, already mentioned, appears to have been based on the conviction that the process of finding good axioms was a dubious enterprise. Note that logic in itself was not something that was shunned in India; without going into a detailed discussion of Indian systems of logic, it is enough to note here that time and again Indians use deductive logic to demonstrate inconsistencies or to refute the positions of an adversary in debate, rather than to derive what western cultures have long sought through that method - namely, certain truth.

This attitude of computational positivism had actual practical implications for the predictive methods used. For example, it is now known that Indian astronomers used epicycles with time varying parameters or patched ellipses, both of which would not have been considered suitable by somebody reared in Greek schools of thought, because such devices did not conform to their ideas of perfect figures, symmetry, beauty etc. One may summarise the situation by saying that while the Indian astronomer found the epicycle a useful tool of representation, he would cheerfully abandon the classical epicycle if he found something which was more efficient or led to a shorter algorithm and to better agreement with observation. For somebody subscribing to Greek ideals, however, this attitude would presumably seem sacrilegious - the rejection of the epicycle would question the basic assumption that the circle is a perfect figure preferred by nature, and hence precipitate a major philosophical crisis.

III - Playfair's perplexities:

The Historical Turning Point

The first critical examination of the Indian knowledge system in astronomy from direct sources appears in a fascinating review made by John Playfair (1748-1819), in a paper called *The Astronomy of the Brahmins* published in the Transactions of the Royal Society of Edinburgh in 1790. The paper is particularly interesting because of the time at which it was written. Towards the end of the 18th century, British power had expanded sufficiently in India that curiosity grew about Indian knowledge systems and access to Indian scholars familiar with the original texts became feasible. Such a review would not have been possible much earlier because British political and military presence in India was insignificant. On the other hand, by the 1830s European knowledge systems had become so evidently superior and British power in India so dominating, that Indian knowledge systems came to be treated as useless.

Playfair was a mathematician, physicist, geologist, astronomer and historian, and held the positions of Professor of Mathematics (1785) and Professor of Natural Philosophy (1805) at the University of Edinburgh. He appears to have been the first British professional mathematician who seriously examined classical Indian mathematics.

His 1790 paper is characterised by wonder and puzzlement. He finds that the predictions made by Indian astronomers were always better than those made by Ptolemy, who had continued to remain the authority in Europe till the advent of Newton. But furthermore he finds that the accuracy of Indian predictions often matched those that were being made in Europe at his time. Playfair represented an age when British science was full of post-Newtonian confidence, and is therefore reduced to speculating about the reasons for the extraordinary accuracy of an apparently non-Newtonian system. He considers three hypotheses. The first is that it is pure coincidence. But as he finds excellent agreement on nine different astronomical parameters, he is forced to abandon this hypothesis. The second hypothesis is that

...Some ages ago, there had arisen a Newton among the Brahmins, to discover that universal principle which connects, not only the most distant regions of space, but the most remote periods of duration; and a Lagrange, to trace, through the immensity of both, its most subtle and complicated operations.

This hypothesis he again finds difficult to accept, so he is left with only one final option, namely, that 'Observations made in India when all Europe was barbarous or uninhabited, and those ... made in Europe 5,000 years afterwards ... come in mutual support'. He remarks repeatedly in his paper on the 'wonderful certainty and precision' of the 'ingenious', 'extra-ordinary' and 'extremely simple' rules that the Indians used. At the same time he also notes that 'the Brahmins follow its rules without understanding its principles ... give no theory... [are] satisfied with calculation' .

Eight years later Playfair returned to his examination of Indian mathematics, this time writing on trigonometry (Playfair 1798); and his reactions on this subject once again are very similar to his earlier ones on astronomy. Scrutinising the trigonometry of the *Surya-Siddhanta,* he first notes how the book starts with 'extravagant fiction' (such as for example how a man's life extended to a thousand years in some previous golden age), but also notes that this 'singular book' contains

... a very sober and rational system of astronomical calculation; and even the principles and rules of trigonometry, a science of all others the most remote from fable, and the least susceptible' of poetical decoration.

He then, goes on to decipher what we would today call an algorithm for computing the trigonometric sines, and the rule as he traces it is as follows.

The numbers in a table of sines constitute a series, in which every term is formed exactly in the same way, from the two preceding terms, viz by multiplying the last by a certain constant number, and subtracting the *last but* one from the product.

In modern mathematics this can immediately be seen as a second order difference equation where, in a table of sines, the *n*th entry is linearly related to the two previous entries, namely n-1 and n-2. A little manipulation of the rule shows that it is actually solving what we would today call a standard finite difference approximation to the well-known second order differential equation whose solution is the sine function. The algorithm therefore is basically an approximate method of integrating the appropriate differential equation by a finite difference scheme that is extremely easy to remember. As Playfair remarks, 'by its help one might at any time compose for himself a complete set of trigonometric tables in a few hours, without the assistance of any book whatever.' A table of 24 sines, of the kind that Aryabhata listed in 499 CE, would actually be a matter of a few minutes' calculation.

It is clear from these examples that, by methods that Western scholars could not recognise as late as 1800, Indian astronomers were achieving accuracies that were considered astonishing. This inference is confirmed by the opinion of other scholars who have not always taken a generally admiring view of Indian scientific achievements. For example, David Pingree (2001) says,

The way medieval Indian astronomers and mathematicians pursued science was demonstrably different and in many ways far more successful than the way in which medieval Europeans pursued astronomy and mathematics.

David Bressoud (2002), in a paper titled *Was calculus invented in India?* (his answer is no), points out that Indian astronomers (who had learnt of earlier Greek accomplishments) made such 'conceptual breakthroughs that would allow them to reduce the errors [from 1 in 10^4] to 1 in 10^{12}.' A legitimate question therefore is how this was accomplished; what was the approach or philosophy that enabled Indians to do so well without the use of Newtonian dynamics?

IV - Computational Positivism

The arguments in the previous two sections have shown how classical Indian astronomy achieved levels of accuracy that were considered astonishing in late 18th century Europe by methods which place computation and observation at the forefront. These methods can be legitimately criticised as lacking a model or a theory of astronomical phenomena. The question can therefore be asked whether the philosophy of computational positivism makes any sense. 'There can be no doubt that for more than a thousand years it was very effective. One could even argue that it was informed by a certain minimalism, in the sense that it avoided making any (unnecessary) hypotheses. It was inspired by an appreciation of the extraordinary power of computation. It avoided the kind of excesses that axiomatist arguments often produced in the West. On this last point, we must note that the remarkable deductionist successes of Euclidean geometry encouraged generations of Western scholars to attempt similar methods in a wide variety of other fields - not only astronomy but also even religion and philosophy. Euclid himself, in another of his works called *The Phaenomena,* claimed to 'prove' that the earth was in the middle of the cosmos; Ptolemy similarly believed that each planet had made a voluntary decision to behave according to the system laid down in his book - thus opening the possibility of a great philosophical and moral crisis if observations were to disagree with his model. (Indeed there are charges that Ptolemy some times manipulated observational data so that they could agree with his predictions.) There were other Greek mathematicians who 'proved' that the moon is more than half of the size of the earth. It was such excess, and a tenuous link to observation, that made Greek philosophers the butt of the scathing criticism of Francis Bacon, who for example said of Aristotle that he was a quack, 'composing a manual of madness that made us slaves of the word.' It is not difficult to see that when axiomatist-deductionist arguments draw egregious conclusions, scientists may at some stage turn to the 'cleaner' and more certain world of computation and to strict comparison with observation, to avoid what Bacon called 'spinning idle theories of causation'.

V - The Newtonian Synthesis

If in the early 19th century European astronomy quickly overtook Indian in accuracy of prediction, the reasons are to be sought in the extraordinary revolution that was wrought by Newton. It is therefore of interest to consider exactly how Newton was able to achieve such accuracies. Surely, if there were an Indian counterpart of Playfair, let us say towards the middle of 19th century, he would have been as perplexed by the extraordinary successes of Newtonian mechanics as Playfair was by those of classical Indian astronomy. Newton's work revealed the limitations of computational positivism, but interestingly by combining the power of axiomatist and algoristic approaches.

In the literature on the European scientific revolution (e.g., Rossi 2001), there is usually considerable discussion on the developments preceding Newton in Europe, with particular emphasis on Copernicus, Kepler and Galileo. While these three figures were undoubtedly responsible for very significant developments, what would have puzzled the Indian Playfair most would be the work of Newton as embodied in the *Principia Mathematica Philosophiae Naturalis (The Mathematical Principles of Natural Philosophy).* For here was a treatise in which what are claimed to be universal laws of motion are stated in the form of three axioms, a vast variety of results are deduced (by pure logic) from these axioms, and then, from an analysis of observational data that had become available by Newton's time, the principle of gravity is proposed. It is shown that these four are enough to describe the motion of the planets, the appearance of comets, the consistency of the acceleration due to gravity at the earth's surface with the orbital motion of the moon, and the periodicities of the tides. The methods that Newton used were remarkable, because they represented a powerful synthesis of the two different approaches to exact science we have been discussing. The *Principia,* which embodies these results, divides into two distinct parts. The first, consisting of Books 1 and 2, basically states the three laws of motion, and is written in a severely deductionist style, complete with the Euclidean apparatus of axioms, propositions, corollaries, theorems, QED and so on. The character of the treatment changes dramatically in Book 3, which is often algoristic: i.e., specific, precise instructions regarding calculations to be made are set out - in the imperious style that is familiar in Indian astronomical texts such as the *Aryabhatiya.* Indeed, Newton finds it necessary to insert a section called *The rules of philosophical reasoning* before presenting Book 3, for he clearly saw that the conclusions he wanted now to draw could not be done in the Euclidean style he had adopted in the first two books. Instead he appeals again and again to his own rules of philosophical reasoning, for here he is no longer doing deductionist mathematics; rather he is assessing observations and trying to make propositions whose numerical consequences can be compared with those observations for support. For example, what is today often called the universal law of gravitation appears as Proposition 7 in Book 3. Newton clearly hesitated to call it a law, for he had serious philosophical doubts about whether it merited the status. These views were discussed in an extensive correspondence he had with the Rev Doctor Bentley; and Newton admits here that the status of his work on gravitation has to be seen as being different from that on his three laws of motion. It was in connection with gravitation that Newton famously said in the *Principia,* 'I frame [often translated from the Latin as feign] no hypotheses; for whatever is not deduced from the phenomena is to be called an hypothesis; and hypotheses, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy. In this philosophy particular propositions are inferred from the phenomena...' (Motte 1995, 442-443).

There has been considerable discussion about Newton's style in the *Principia.* The general view is that although he expounded his work by Euclidean methods, he actually derived his results by a mixture of calculus and algebra, dressing them up in geometry because that is what in his view the 'dignity' of the subject demanded. (This question of 'dignity' comes up again and again in Chandrasekhar's (1995) recent commentary on the *Principia,* but Chandrasekhar himself differs from the standard view and considers that Newton did indeed do it in the Euclidean style.) There is no doubt that Newton's thinking was greatly influenced by the work of Descartes (1596-1650), in particular his algebraisation of geometry. This Cartesian project was a turning point in the history of the scientific revolution in Europe, for it was responsible for a crucial advance in the so-called mathematisation of science that characterised the revolution. Descartes's *Discourse de La Methode* contained a kind of appendix titled *La Geometrie,* published in 1637. One of the remarkable things about this book is the declaration of Descartes at the end of its first paragraph,

And I shall not hesitate to introduce these arithmetical terms into geometry, for the sake of greater clearness.

'Clearness' was thus the goal of the project. After noting that 'it is sufficient to designate each [line, with its length] by a single letter' like *a, b* etc, rather than the BD and GH that appear in a figure accompanying the text (and as they still do in texts of elementary geometry), Descartes went on to say

Here it must be observed that by a^2 [which he often wrote as *aa*], b^3, and similar expressions, I ordinarily mean only simple lines, which, however, I name squares, cubes, etc, so that I may make use of the terms employed in algebra.

The translators point out, in a footnote,

At the time this was written, a^2 was commonly considered to mean the surface of a square whose side is *a,* and *b* to mean the volume of a cube whose side is *b;* while b^4, b^5, ... were unintelligible as geometric forms.

The profound effect on the West of Descartes' algebraised geometry can be gauged by the assessments of John Stuart Mill (who called it 'the greatest single step ever made in the progress of the exact sciences') and Jacques Hadamard (according to whom Descartes 'revolutionised the entire conception of the object of mathematical science'). Incidentally it is interesting to note here that this 'revolutionary' change that Descartes made in Western thinking, looking upon the process of exponentiation as not necessarily dimension enhancing, was something which had come so easily to Indians much earlier. For example *Aryabhatiya* says (ii 3)

Varga is the area of an equal-sided quadrilateral [with equal diagonals] and also the product of two equals; *Ghana* is the product

of three equals and also [the volume of] a twelve-edged solid [of equal sides and diagonals].

Indeed the Indian mathematician Madhava (1349-1425) had already produced, more than two hundred years before Descartes and three hundred years before Maclaurin, infinite series for several trigonometric functions (e g, Roy 1990, Bressoud 2002), containing arbitrarily high powers of the argument (i.e., the angle) of those functions. There is no evidence that Madhava or his distinguished disciples agonised over the number of dimensions of the space that those high \ powers would have implied for a Hellenist mathematician.

It is well known that Newton was inspired by Descartes's algebraised geometry. For example an account by Newton's nephew (perhaps rather romanticised, according to Gleick 2003), says:

He [Newton] then young as he was took in hand Des-Cartes's geometry (that book which Descartes in his Epistles with a sort of defiance says is so difficult to understand). He began with the most crabbed studies and books, like a high spirited horse who must be first broke in crabbed grounds and the roughest and steepest ways, or could otherwise be kept within no bounds. When he had read two or three pages and could understand no farther he being too reserved and modest to trouble any person to instruct him be gain again & got over three or four more till he not only made himself master of the whole ... but [even] discovered the errors of Descartes...

One well-known Newton scholar (Gleick 2003) says

...the thick wad of Newton's research papers surviving from the later months of 1664 stand firm witness that it was indeed from the hundred or so pages of the *Geometrie* that his mathematical spirit took fire…Above all, I would assert, the *Geometrie* gave him his first true vision of the universalising power of the algebraic free variable, of its capacity to generalise the particular and lay bare its inner structure.

We can therefore say that the novelty and power of the new European approach to the exact sciences arose from a combination of the methods of geometry and algebra. This leap was not made by Galileo, who is often credited with the mathematisation of science, did not make this leap: because for him mathematics was still basically geometry. As he said (e g, Gleick 2003)

... The book [of nature] ... is written in the language of mathematics. ... Its characters are triangles, circles, and other geometrical figures, without which it is humanly impossible to understand a single word of it...

Note the conspicuous absence of number in that great book!

Indeed the 'mathematisation' of science that is so celebrated as a characteristic of the European scientific revolution fails to recognise that Ptolemy (and others before him in Greece, Babylonia and elsewhere) had already used mathematics in science in their own ways. What was really new was a synthesis of Greek geometry and what Descartes called the 'barbarous' art of Indo Arabic algebra. It is in this sense that the remark of Hermann Weyl (1929) is appropriate:

Occidental mathematics has in past centuries broken away from the Greek view and followed a course which seems to have originated in India and which has been transmitted, with additions, to us by the Arabs; in it the concept of number appears as logically prior to the concepts of geometry.

To summarise, therefore, the long run of success of computational positivism came to an end with Newton, who brilliantly fused classical Greek axiomatism with the new Indo-Arabic tools of algebra and algorithm, inspired by the efforts of integrators like Descartes.

VI - Equations and Computation

Although we now easily talk of Newton's laws and his equations of motion, and we know of the inf1uence that algebra had on him, it is paradoxical that Newton himself wrote very few equations in the *Principia;* in particular the celebrated equation $F = ma$ does not appear in the book. Newton's reasoning was expressed in closely argued, convoluted and continuous prose, and has in fact been hard to follow for most readers (now as at the time of its publication). Today, however, the standard first encounter with Newtonian mechanics, say for a young student, is through equations, algebra and calculus rather than by geometric Euclidean arguments. The commentary that Chandrasekhar wrote for the 'common reader' might in fact be said to translate Newton's geometrical reasoning into more easily understood algebra. And the form in which Newton's equations are written today owe a great deal to the prodigious but under-rated genius of Leonhard Euler (1707- 1783)

In the classical eastern algoristic tradition, however, equations in some form or the other had appeared much earlier. For example Hayashi (1995), in his important work on the Bakhshali manuscript, shows how equations were written at the time the manuscript was composed - his best estimate for the date is about 800 CE. It therefore appears that the tradition of algorithm, algebra and equations are tied together, and formed one composite system that has introduced new power to the world of physical sciences over the least three centuries, as Hermann Weyl implied in the quote above. We may look upon an equation as a sentence in an artificial language, expressing a relation among symbols that represent various physical or mathematical quantities. There are excellent reasons why equations are so powerful. In the first place an equation has clarity and precision. At the same time non-linear equations can have multiple solutions - something that was realised long ago in the work of Brahmagupta (b 598 CE), for example, who solved an indeterminate quadratic equation, which has infinitely many solutions. The equation therefore is a powerful, precise and clear statement that can embody multiple possibilities. It is a sutra in a new, artificial language.

The advent of the transistor and the large-scale integrated circuit in the second half of the 20th century may be resulting in another revolution. First of all it has led to computers of ever-increasing power: according to what professionals in the field call Moore's Law, a measure of this power has been doubling every eighteen months for nearly three decades now, and seems set to do so for at least another decade. The fact that an enormous number of calculations can now be made so inexpensively and rapidly is changing the way we look at the world. In the early years of the computer era, it was seen as a powerful slave that mechanically carried out instructions given by an intelligent human being. In the 70s and 80s scientists began to see the computer as a powerful ally that they could work with to explore, test or exploit their intuitions about nature. (Indeed, as this view gained more adherents, there were many who warned that computers may ruin science; see e.g., Truesdell 1984.) In more recent decades it looks as if the computer is slowly becoming an agent of discovery. The first field, which benefited from this new power, was the science of non-linear dynamical systems. It was really the advent of the computer that made it possible to solve systems of non-linear ordinary differential equations and to begin to understand how chaotic behaviour can emerge even in relatively simple and familiar systems. (In brief we may say that non-linearity represents a situation where the output of a system is disproportionately large or small compared to the input.) The first studies of chaotic behaviour in what otherwise had appeared to be perfectly predictable, deterministic systems has changed the way that we look

at the concept of randomness. These studies showed that even classical Newtonian systems had limited predictability horizons, and systems that exhibit such behaviour are not necessarily strange and exotic: the pendulum, the string and even the planetary system can be chaotic. Indeed, the following simple algorithm generates chaos.

(1) Choose almost any number between 0 and 1 as input.
(2) Multiply it by 4 times its complement from 1 to get the second number.
(3) Using this number as the new input, compute the third number by repeating step 2.
(4) Generate as long a sequence of numbers as desired by repeated applications of step 2.

The sequence so generated is 'chaotic' in the sense that it will appear completely random. Each number in the sequence depends on the previous one in a completely deterministic way, but nevertheless the sequence is chaotic, as pairs of successive numbers are uncorrelated in the statistical sense. The study of such systems has grown explosively in the last few decades, largely because of the availability of powerful computers (there has been much interesting theory as well). The change that the possibility of such 'deterministic' chaos has wrought in our views of the connection between determinism and Newtonian dynamics is so profound that Sir James Lighthill, occupant of the same prestigious Cambridge chair that Newton had held 300 years earlier, went so far as to render a public apology on behalf of the scientific community of practitioners of mechanics for having misled the world at large into such / fundamentally incorrect perception of Newtonian dynamics (Lighthill 1986).

The example given above is just one instance where computers are changing the way we look at nature. There are others involving many interesting kinds of computer simulations with implications even in the social sciences. It is perhaps therefore no surprise that a school of thought is now emerging that looks at computation as fundamental to our understanding of the universe. This view has been expressed in a very strong and forceful way by Stephen Wolfram (2002) in a recent book. We have noted above that the possibility of describing nature based on equations revolutionised scientific thinking in the last few centuries; Wolfram now claims that a new kind of science is currently emerging, 'based on the much more general types of rules that can be embodied in simple computer programs'. The basis for this assertion is that it is possible to devise absurdly simple algorithms which result in system behaviour that can be extraordinarily complex -as complex as may be exhibited by any set of very complicated equations (we saw one example already). Wolfram demonstrates this property using what are known as cellular automata. Here both time and space are discrete, and an automaton in each spatial cell updates itself from its state at any given time to a new state after the specified time step. The result of such operations can be presented in the form of what I call a 'quilt' (with colour in each cell denoting its state). The quilt shows the evolution of the system in a diagram in which cell location is along one axis and time on the other. It turns out that such quilts can be either simple checkerboards or have intricate nested patterns, or exhibit strange mixtures of 'order in one region and apparent randomness in another region, or be random everywhere. The wealth and diversity of complex behaviour that results from these simple algorithms can be breathtaking. This kind of study leads Wolfram first to assert that all processes, whether they are produced by human effort or occur spontaneously in nature, can be viewed as computations. Indeed he goes so far as to say that the universe may be just 'a simple programme, which, if run long enough, would reproduce our universe in every detail.'

Current scientific opinion does not in general agree with Wolfram for a variety of reasons, many of which are indeed sound. Nevertheless there is the definite possibility that the 'lies' of the cellular automaton in an artificial Boolean micro-world can lead' to useful truths in the real, physical macro-world. There are some areas, such as, for example, certain aspects of the problem of phase transition in physics, where cellular automata might provide answers that are applicable to the real world. But my point here is not to take a position on whether a simple programme exists that will reproduce our universe, but only to say that that is a possibility that at least some scientists will no longer dismiss as absurd. This suggests that some form of computational positivism is beginning to be seriously considered once again in the world of science. It is of course very hard at present to see how the present powerful web of theory, law and algorithm can be replaced, but that the computer will raise fundamentally new questions seems certain.

VII - Conclusion

I have argued in this essay that even in the approach to an exact science like astronomy, where the data available to different civilisations in the world have been virtually uniform, the systems of thought that have evolved in the effort to organise the reality of those observations into quantitative, predictive schemes have not been universal. At least two different scientific cultures can be identified: one that depends on devising a system of axioms and deducing through pure logic conclusions that may be expected to represent nature and another where one devises algorithms that will produce results in agreement with observations. One may call these two cultures respectively as that of model makers and algorisers. Model making in this sense appears to have been derived from Greek traditions, and has strongly influenced scientific thinking in the West. On the other hand in India, and apparently in Babylonia as well (which we have not discussed here at all), the approach was that of largely model free algorising. This latter approach may be said to have believed in what I have here called computational positivism. Three hundred years ago in Europe these two cultures seem to have come together, in particular in the work of Descartes who married algebra and geometry and of Newton, who wrote his great epoch making book using a mix of Euclidean and algoristic approaches, In succeeding centuries the mathematical equation has played a big role, for it provided a language in which a sentence could at one and the same time be clear, quantitative and precise, but also permit multiple solutions and even exhibit chaotic behaviour. But this understanding has in fact become possible only with the advent of computers, and there is increasing evidence that as their power reaches ever-higher levels, the ideas behind computational positivism may not have been entirely abolished by Newtonian triumphs. It is of course not possible to predict what will happen in the future, but the recognition that there are two cultures in a subject that, in popular perception, is as universal as mathematics, is something that helps to understand at once how different civilisations have approached this problem in different ways, and how there might be multiple routes to generating what may be seen as scientific knowledge.